Comparisons of numerical experiments about GRNMM methods

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Abstract

Based on the global relaxed non-stationary multisplitting multi-parameter (GRNMM) methods, we give comparisons of numerical experiments about GRNMM methods and show the efficiency of GRNMM methods associated with TOR multisplitting for solving a large sparse linear system whose coefficient matrix is an H-matrix.

Keywords: Relaxed non-stationary multisplitting multi-parameter method; Parallel multisplitting; Global relaxed method; H-matrix

1 Introduction

For solving the large sparse linear system

$$Ax = b, \tag{1}$$

where $A \in R^{N\times N}$ is a square nonsingular H-matrix and $x, b \in R^N$, an iterative method is usually considered. The concept of multisplitting for the parallel solution of linear system was introduced by O'Leary and White [1] and further studied by many authors [1-23]. The multisplitting method can be thought of as an extension and parallel generalization of the classical block Jacobi method [4]. In this paper, we give comparisons of numerical experiments about GRNMM methods and show the efficiency of GRNMM methods associated with TOR multisplitting for solving a large sparse linear system whose coefficient matrix is an H-matrix.

2 GRNMM method

Global Relaxed Non-stationary Multisplitting Multi-parameter TOR mthod (GRNMM-TOR) was proposed in [23]. The algorithm is as follows.

Algorithm 1 (GRNMM-TOR)

Given the initial vector
For $m = 0, 1, \ldots,$ *repeat* (I) *and* (II), *until convergence.*
In k processors, $k = 1, \ldots, \alpha$, *let* $y_k^{(0)} = x^{(m)}$.

(I) *For* $i = 1, 2, \ldots, q(m,k)$, (*parallel*) *solving* $y_k^{(i)}$.

$$[D - (\alpha_k L_k + \beta_k F_k)]y_k^{(i)} = [(1-\gamma_k)D + (\gamma_k - \alpha_k)L_k + (\gamma_k - \beta_k)F_k + \gamma_k U_k]y_k^{(i-1)} + \gamma_k b.$$

(II) *Computing*

$$x^{(m+1)} = \omega \sum_{k=1}^{\alpha} E_k y_k^{(q(m,k))} + (1-\omega)x^{(m)}.$$

The Algorithm 1 can be written as:

$$x^{(m+1)} = H^{(m)}(\alpha_k, \beta_k, \gamma_k, \omega)x^{(m)} + G^{(m)}(\alpha_k, \beta_k, \gamma_k, \omega)b, m = 0,1,..., \tag{2}$$

where

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$$H^{(m)}(\alpha_k,\beta_k,\gamma_k,\omega)=\sum_{k=1}^{\alpha}E_k\{[D-(\alpha_kL_k+\beta_kF_k)]^{-1}[(1-\gamma_k)D + (\gamma_k-\alpha_k)L_k+(\gamma_k-\beta_k)F_k+\gamma_kU_k]\}^{q(m,k)}+(1-\omega)I,$$

$$G^{(m)}(\alpha_k,\beta_k,\gamma_k)=\omega\sum_{k=1}^{\alpha}E_k\left\{\sum_{i=1}^{q(m,k)-1}[D-(\alpha_kL_k+\beta_kF_k)]^{-1}[(1-\gamma_k)D + (\gamma_k-\alpha_k)L_k+(\gamma_k-\beta_k)F_k+\gamma_kU_k]^i\right\} \times[D-(\alpha_kL_k+\beta_kF_k)]^{-1}\gamma_k.$$

3 Numerical comparisons

In this section, we present some numerical experiments which compare the performance of GRNMM-TOR methods with LRNMM-AOR and LRNMM-SOR methods, and numerical experiments achieve effective improvement compared with the methods in [6,10]. By using difference discretization of partial different equation, we can obtain the corresponding coefficient matrix form of the linear system (n= 6), which is as follows.

$$A=\begin{bmatrix}4&-1&0&0&0&0\\2&2&1.5&0&0&0\\0&1&3&-1&0&0\\0&0&1.5&2&2&0\\0&0&0&1&4&-1\\0&0&0&0&2&2\end{bmatrix},b=\begin{bmatrix}3\\5.5\\3\\5.5\\4\\4\end{bmatrix}.$$

$$E_1=diag(1,1,0,0,0,0),E_2=diag(0,0,1,1,0,0),E_3=diag(0,0,0,0,1,1).$$

$$L_1=\begin{bmatrix}0&0&0&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0.3&0&0&0\\0&0&0&1&0&0\\0&0&0&0&-0.8&0\end{bmatrix},L_2=\begin{bmatrix}0&0&0&0&0&0\\-1.5&0&0&0&0&0\\0&0.5&0&0&0&0\\0&0&0.7&0&0&0\\0&0&0&0.5&0&0\\0&0&0&0&0.4&0\end{bmatrix}.$$

$$L_3=\begin{bmatrix}0&0&0&0&0&0\\0.6&0&0&0&0&0\\0&1.5&0&0&0&0\\0&0&-1.2&0&0&0\\0&0&0&0.3&0&0\\0&0&0&0&0.8&0\end{bmatrix},U_k=D-L_k-A,k=1,2,3.$$

$$L_1=\hat{L}_k+\hat{F}_k=\begin{bmatrix}0&0&0&0&0&0\\0.3&0&0&0&0&0\\0&0.7&0&0&0&0\\0&0&0.1&0&0&0\\0&0&0&0.9&0&0\\0&0&0&0&-0.5&0\end{bmatrix}+\begin{bmatrix}0&0&0&0&0&0\\0.7&0&0&0&0&0\\0&0.3&0&0&0&0\\0&0&0.2&0&0&0\\0&0&0&0.1&0&0\\0&0&0&0&-0.3&0\end{bmatrix}.$$

$$L_2=\hat{L}_k+\hat{F}_k=\begin{bmatrix}0&0&0&0&0&0\\-0.7&0&0&0&0&0\\0&0.3&0&0&0&0\\0&0&0.2&0&0&0\\0&0&0&0.4&0&0\\0&0&0&0&0.2&0\end{bmatrix}+\begin{bmatrix}0&0&0&0&0&0\\-0.8&0&0&0&0&0\\0&0.2&0&0&0&0\\0&0&0.5&0&0&0\\0&0&0&0.1&0&0\\0&0&0&0&02&0\end{bmatrix}.$$

$$L_3 = \hat{L}_k + \hat{F}_k = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 \end{bmatrix}.$$

Let the initial guess and the tolerance be $x^{(0)} = (10,30,-20,-40,-8,9)^T$ and $\varepsilon = 10^{-10}$, respectively.

By numerical experiments, the results of performance improvements with GRNMM-TOR method and LRNMM-SOR method are shown in Table 1. Furthermore, ρ_{opt} and ite_{opt} denote spectral radius of approximate optimization and iterative numbers of approximate optimization, respectively. The improvements percentage % are obtained from $1-\frac{ite_{opt(\mathrm{Re}f)}}{T_{opt(thispaper)}}$. Similarly, the performance improvements results with GRNMM-TOR method and LRNMM-AOR method are shown in Table 2.

Let the initial guess and the tolerance be $x^{(0)} = (0,10,-20,20,30,-30)^T$ and $\varepsilon = 10^{-10}$, respectively.

The performance improvements results with GRNMM-TOR method and LRNMM-SOR method are shown in Table 3. Similarly, the performance improvements results with GRNMM-TOR method and LRNMM-AOR method are shown in Table 4.

TABLE 1 Comparison of improvements percentage

method	ρ_{opt}	ite_{opt}	improvements performance%	Ref
LRNMM-SOR	0.6433	62	0	[16]
GRNMM-TOR	0.5705	49	20.97%	this paper

TABLE 2 Comparison of improvements percentage

method	ρ_{opt}	ite_{opt}	improvements performance%	Ref
LRNMM-AOR	0.6066	53	0	[10]
GRNMM-TOR	0.5705	49	7.55%	this paper

TABLE 3 Comparison of improvements percentage

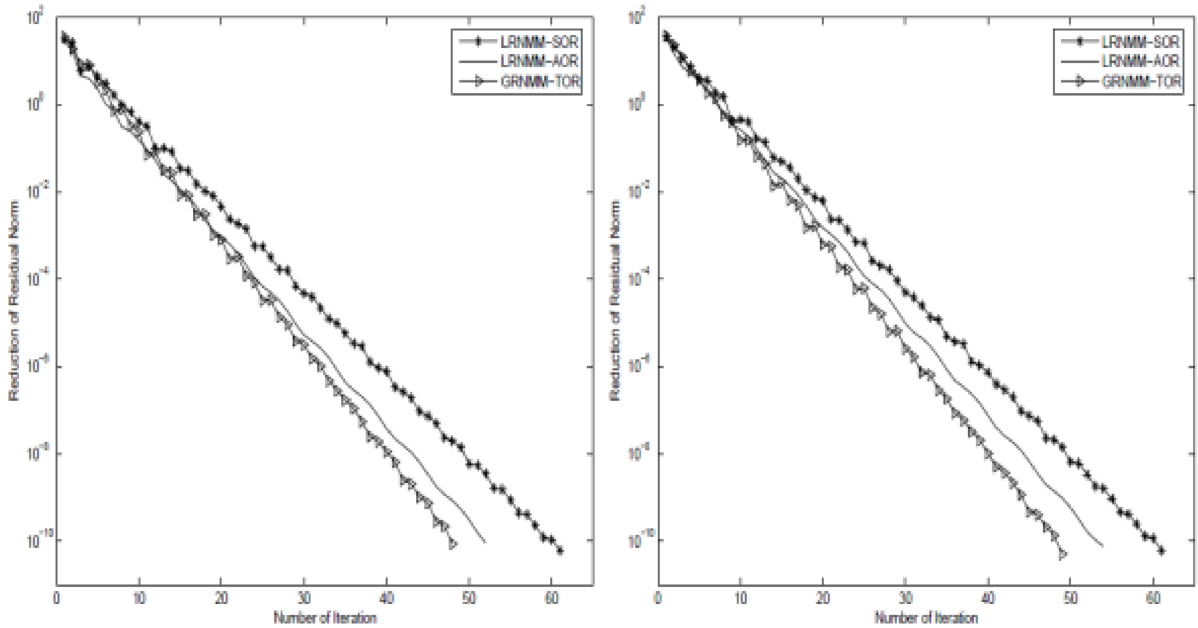
method	ρ_{opt}	ite_{opt}	improvements performance%	Ref
LRNMM-SOR	0.6433	62	0	[16]
GRNMM-TOR	0.5705	50	19.35%	this paper

TABLE 4 Comparison of improvements percentage

method	ρ_{opt}	ite_{opt}	improvements performance%	Ref
LRNMM-AOR	0.6066	55	0	[10]
GRNMM-TOR	0.5705	50	9.09%	this paper

In Figure 1, we show the detailed comparison of residual norm decline about three methods. From Figure 1, we may see clearly that GRNMM-TOR method can achieve much faster convergent speed than LRNMM-AOR method and LRNMM-SOR method.

Remark 3.1 The above numerical experiments indicate: By using our methods, we really achieve effective improvement compared with LRNMM-AOR method and LRNMM-SOR method. When comparing with LRNMM-SOR method and LRNMM-AOR method, the number of iterations for convergence of GRNMM-TOR method improved 20% and 10%, which the tolerance for convergence is residual norm less than $\varepsilon = 10^{-10}$.



FIRURE 1 Comparison of reduction of residual norm with GRNMM-TOR, LRNMM-AOR and LRNMM-SOR

4 Conclusions

In this paper, based on the global relaxed non-stationary multisplitting multi-parameter TOR iterative methods for solving linear systems of algebraic equations Ax=b, we show the efficiency of GRNMM methods associated with TOR multisplitting for solving a large sparse linear system whose coefficient matrix is an H-matrix. Furthermore, efficiency of the global relaxed non-stationary multisplitting multi-parameter methods are shown by numerical experiments. Numerical experiments show that when choosing the approximately optimal relaxed parameters, GRNMM-TOR methods have faster convergent rate compared with LRNMM-AOR method and LRNMM-SOR method.Further performance improvement, one can consider how to choose the approximately optimal relaxed parameters to reduce the cost of choosing the relaxed parameters and improve performance
strongly.

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